OpenDreamKit Work Package 6
Data/Knowledge/Software-Bases

Michael Kohlhase
FAU Erlangen-Nürnberg
http://kwarc.info/kohlhase

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1 Introduction

Conclusion: What are we doing in WP6 in terms of a VRE

- SageMath/CoCalc and WP6 approach (Math-in-the-Middle; MitM) are both attempts at making a VRE Toolkit.

- overall pattern: design – prototype – scale

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  - Safety: by semantic (i.e. context-aware) objects passed.
  - Extensibility: any open-API system (i.e. with API CDs) can play.
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- Plan for Review Period 3: Extend to external use-cases users (scale and deploy publicly)
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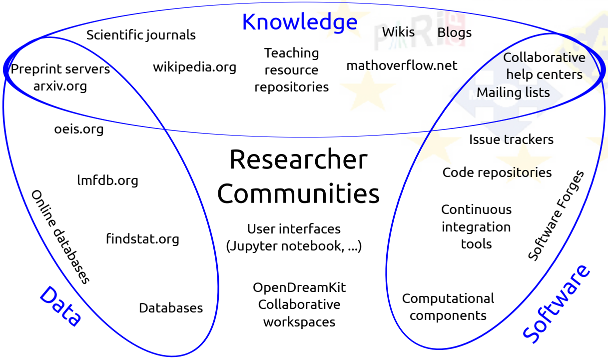
Background: WP6 (Data/Knowledge/Software-Bases)

- From the Proposal:



Storage resources (local, shared folder, cloud)	Computational resources (local, super computer, cloud)

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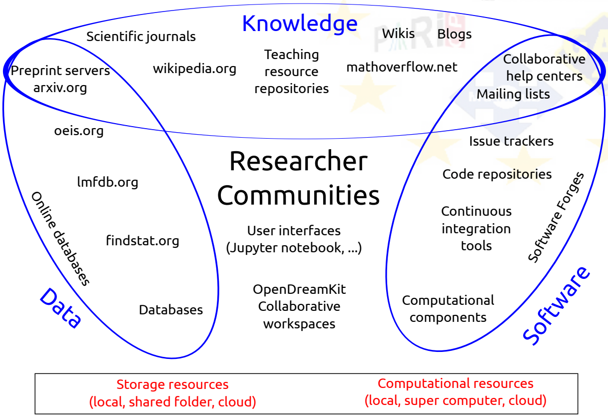

- Proposed Focus: Supply this data to VRE components in an integrated fashion programmatically

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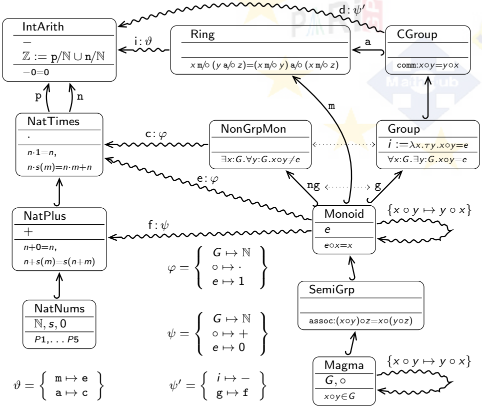

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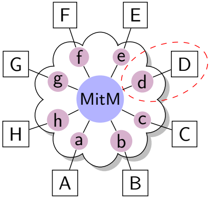

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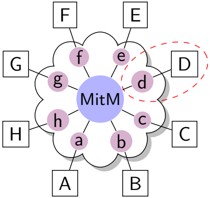

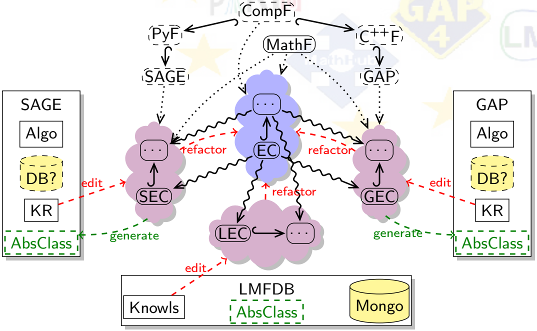

Paper: *Interoperability in the OpenDreamKit Project: The Math-in-the-Middle Approach* [CICM 2016]

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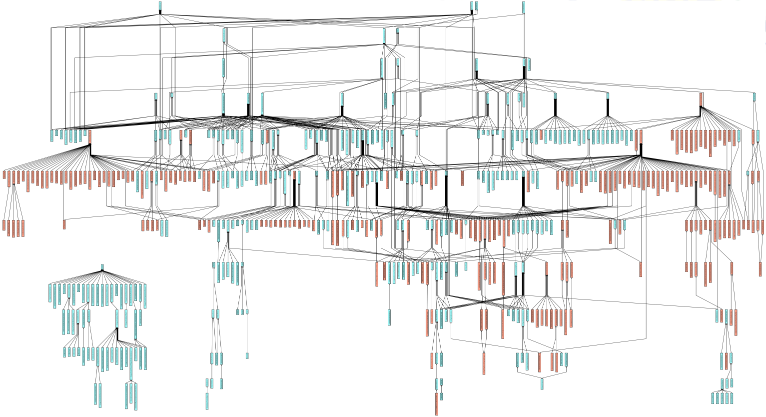

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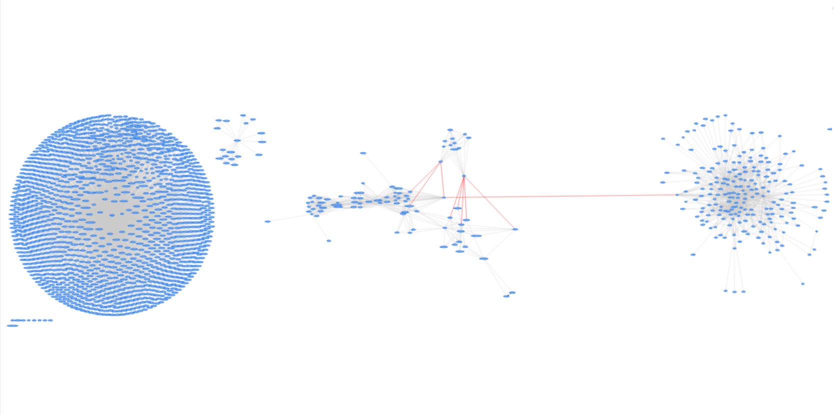
  - WS in Berlin (Feb '17): Math-in-the-Middle Ontology



2 Mathematical Use Cases

Running Example/Use Case: Jane's Invariant Experiments

- Jane wants to experiment with invariant theory of finite groups.
  - She works in the polynomial ring $R = \mathbb{Z}[X_1, \ldots, X_n]$,
  - Goal: construct an ideal I in R that is fixed by a group $G \leq S_n$ acting on the variables, linking properties of G to properties of I and the quotient of R by I.
  - Idea: pick some polynomial p from R and consider the ideal I of R that is generated by all elements of the orbit $O = Orbit(G, R, p) \subseteq R$.
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- Problem: Jane has to learn the GAP and Singular languages and retype the results in them. (error-prone)
- For the sake of example, we will work with $n = 4$, $G = D_4$ (the dihedral group), and $p = 3 \cdot X_1 + 2 \cdot X_2$, but our results apply to arbitrary values.
- Caveat: G is called "D_4" in SageMath but "D_8" in GAP due to differing conventions in different mathematical communities

John's Use Case for LMFDB (slightly abridged)

- John wants to investigate the number fields which are generated by the coefficients of Hilbert modular forms (HMFs).
- LMFDB contains information about all HMFs over base fields F of degree $\mathcal{N} = 2, 3, 4, 5, 6$ (of parallel weight 2 and trivial character).
- Each HMF comes with a Hecke field K which is stored via a defining polynomial (not canonical or minimal $\rightsquigarrow$ difficult to study)
- **Example 2.1.** $K = \mathbb{Q}(\sqrt{2})$ may occur as $x^2 - 2$ and $x^2 - 2x - 1$.
- John would like to be able to
  1. extract these defining polynomials from the LMFDB,
  2. use them to define number fields in SageMath,
  3. find simpler polynomials defining the same fields, and
  4. study their arithmetic properties (e.g., their class numbers).

3 Realizing MitM Interoperability – The Computational Group Theory Case Study –

A MitM Theory in MMT Surface Language

- **Example 3.1.** A theory of Groups

Declaration $\widehat{=}$
name : type [= Def] [# notation]

Axioms $\widehat{=}$ Declaration with type $\vdash F$

ModelsOf makes a record type from a theory.

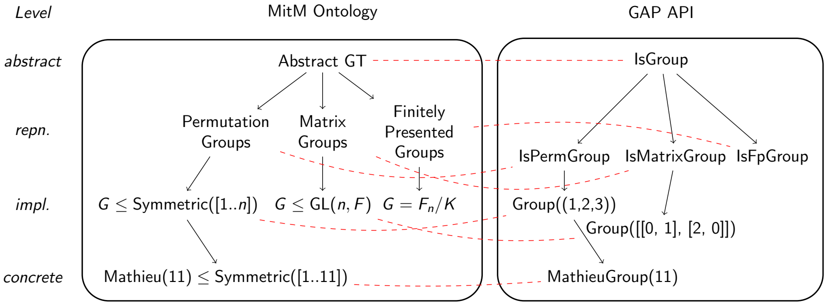
```
theory group : base:?Logic =
  theory group_theory : base:?Logic =
    include ?monoid/monoid_theory ❙

    inverse : U → U ❘ # 1 ⁻¹ prec 24 ❙
    inverseproperty : ⊦ ∀ [x] x ∘ x ⁻¹ ≐ e ❙
  ❚
  group = ModelsOf group_theory ❙
```

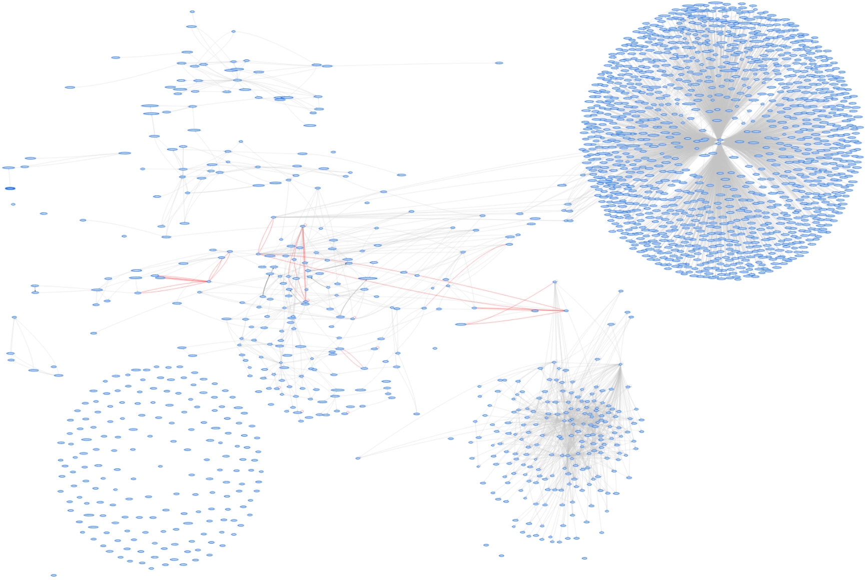
- MitM Foundation: optimized for natural math formulation
  - higher-order logic based on polymorphic λ-calculus
  - judgements-as-types paradigm: $\vdash F \widehat{=}$ type of proofs of F
  - dependent types with predicate subtyping, e.g. $\{n\}\{'a \in \mathrm{mat}(n,n)|\mathrm{symm}(a)'\}$
  - (dependent) record types for reflecting theories

MitM Computational Group Theory

- Four levels of modeling (Following the GAP template)
  - Abstract Level: the group axioms, generating sets, homomorphisms, group actions, stabilisers, orbits, centralizers, normalizers.
  - Representation Level: axiomatizations concrete objects suitable for computation – permutation groups, matrix groups, . . . , also group actions, group homomorphism
  - Implementation Level: permutation groups as subgroups of $S_{\mathbb{N}^+}$, concretely $S_{[1,\ldots,n]}$.
  - Concrete Level: where actual computations happen.
- Alignments between the MitM Ontology and the GAP API



The Knowledge Graph for MitM, SageMath, GAP, Singular



Meaning-Preserving Relations between System Dialects

- **Definition 3.2.** We call a pair of identifiers (a_1, a_2) that describe the same mathematical concept an alignment.
  We call an alignment perfect, if it induces a total, truth-preserving translation.
  (e.g. alignment up to argument order)
- Intuition: Alignments don't need to be perfect to be useful!
  - Alignment up to Totality of Functions (e.g. division undefined on 0 and with $\frac{x}{0} = 0$)
  - Alignment for Certain Arguments (e.g. Addition on natural numbers and addition on real numbers)
  - Alignment up to Associativity (e.g. binary addition and "sequential" addition)

  They still allow for translating expressions between libraries. (under certain conditions)

Jane's Use case in the MitM System

- In SageMath Jane has already built the ring $R = \mathbb{Z}[X_1, X_2, X_3, X_4]$, the group $G = D_4$, the action A of G on R that permutes the variables, and $p = 3 \cdot X_1 + 2 \cdot X_2$.

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- She calls

```
o = MitM.Gap.orbit(G,A,p) # the orbit
i = MitM.Singular(o).Ideal() # the ideal
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- Singular returns the Gröbner base B.
- The MitM server translates B to the SageMath system dialect and sends it to SageMath, where the result is shown to Jane.

$$B = [X_1 - X_4, X_2 - X_4, X_3 - X_4, 5 * X_4].$$

Distributed Computational Group Theory

- Combine SCSCP enabled GAP, SageMath, and Singular with MMT mediator.

Sage

MMT
Mediator

GAP

Singular

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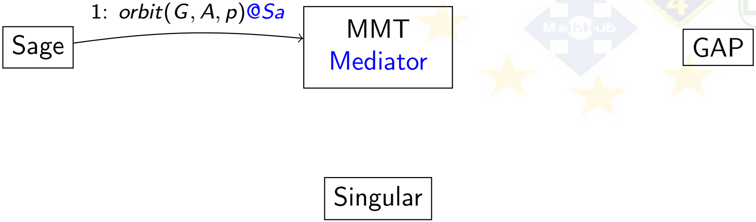

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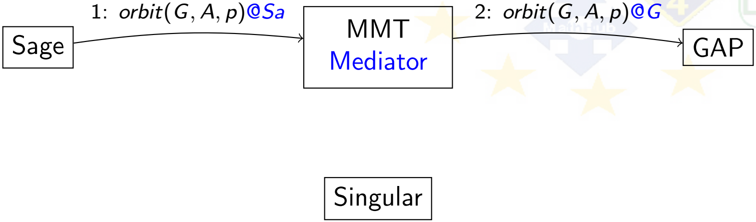

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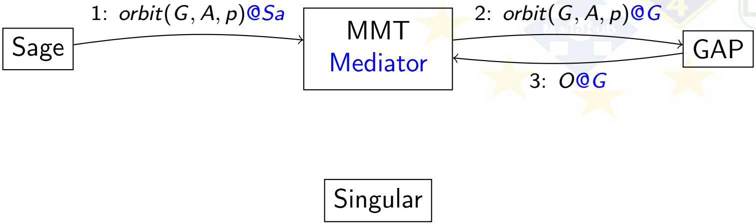

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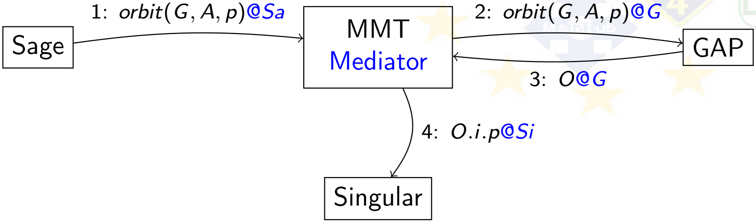

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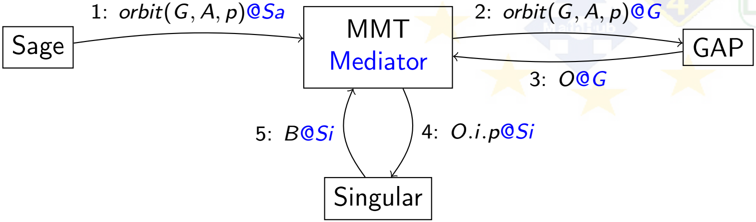

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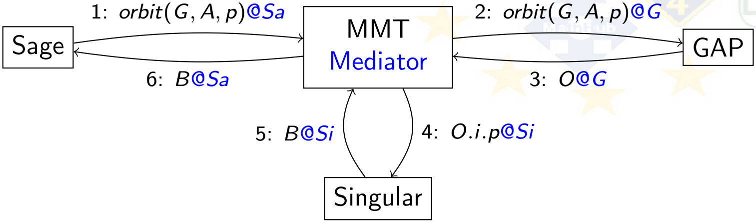

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- Finally, Steve installs a GAP method by calling

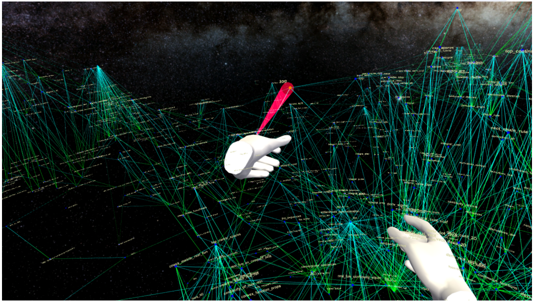
  ```
  InstallMethod(GaloisGroup, "for a polynomial", [IsUnivariatePolynomial],
                p -> MitM("PARIGP", "GaloisGroup", p))
  ```

  $\leadsto$ extends GaloisGroup to rational polynomials in GAP.
- This replaces a significant part of the 1800-LoC radiroot package (by PARI/GP delegation)

MitM-based Integration centers around the MitM Ontology

If you are Really interested in the Graphs

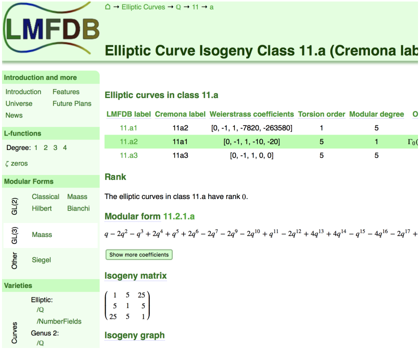
interact with them in 3D

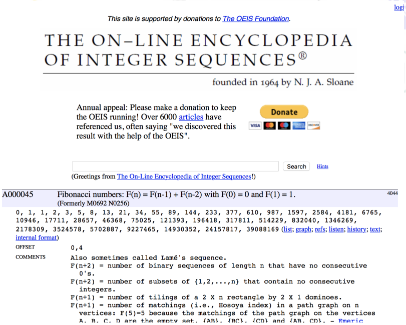


4 MitM InterOperability for Mathematical Databases

Mathematical Knowledge Bases (MKS)

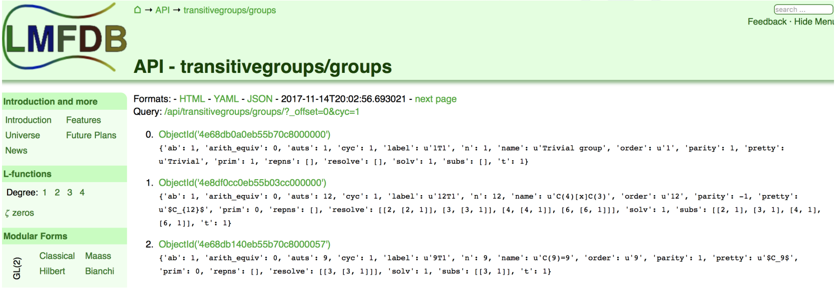
- State of the Art: mathematical object databases (GAP libraries, OEIS, LMFDB)





Mathematical Knowledge Bases (MKS)

- State of the Art: mathematical object databases (GAP libraries, OEIS, LMFDB)
- Problem: human-oriented interface, very limited programmatic API, no computation



- Idea: can't we use MitM Technologies here to integrate?

John's Use Case for LMFDB (slightly abridged)

- John wants to investigate the number fields which are generated by the coefficients of Hilbert modular forms (HMFs).
- LMFDB contains information about all HMFs over base fields F of degree $\mathcal{N} = 2, 3, 4, 5, 6$ (of parallel weight 2 and trivial character).
- Each HMF comes with a Hecke field K which is stored via a defining polynomial (not canonical or minimal $\leadsto$ difficult to study)
- **Example 4.1.** $K = \mathbb{Q}(\sqrt{2})$ may occur as $x^2 - 2$ and $x^2 - 2x - 1$.
- John would like to be able to
  1. extract these defining polynomials from the LMFDB,
  2. use them to define number fields in SageMath,
  3. find simpler polynomials defining the same fields, and
  4. study their arithmetic properties (e.g., their class numbers).

MitM-based Integration of Math Knowledge Bases

- Requirements:
  - a uniformal programatic API to multiple MKB
  - interacting with MKB at the "mathematics Level".
- Idea: use the Math-in-the-Middle Paradigm
  - OMDoc/MMT-based API theories for the mathematical interface ($\leadsto$ MKB records as OM objects)
  - alignments into MitM Ontology (for OM-dialect mediation)
  - extend MMT's built-in query language QMT to general Math query language
- Problems:
  - MKB tables become OMDoc/MMT theories (size problems)
  - how to reconcile MKB records with OMDoc/MMT terms. (encoding/decoding)
  - tow to translate math-level queries to physical database queries

LMFDB Data (Database Level)

- **Example 4.2 (A transitive group represented in in LMFDB).**

```
{
  "ab": 1,
  "arith_equiv": 0,
  "auts": 1,
  "cyc": 1,
  "label": "1T1",
  "n": 1,
  ...
}
```

Legend: for understanding them (LMFDB improved documentation)

- the `cyc` field represents being cyclic (`0` is **false**, `1` is **true**)
- the `n` field represents degree (IEEE Float `1` corresponds to $1 \in \mathbb{N}$)
- ...

Two Problems: that have to be solved for MitM integration

- data base schema is not at the mathematical level (let alone interoperable)
- values are encoded for MongoDB convenience (what do they mean?)

Codecs: Encoding and Decoding Database Values

- **Definition 4.3 (Codec).** A codec consists of two functions that translate between semantic types and realized types.

Codecs

`codec`	:	`type` $\to$ `type`	
`StandardPos`	:	`codec` $\mathbb{Z}^+$	JSON number if small enough, else JSON string of decimal expansion
`StandardNat`	:	`codec` $\mathbb{N}$	
`StandardInt`	:	`codec` $\mathbb{Z}$	
`IntAsArray`	:	`codec` $\mathbb{Z}$	JSON List of Numbers
`IntAsString`	:	`codec` $\mathbb{Z}$	JSON String of decimal expansion
`StandardBool`	:	`codec` $\mathbb{B}$	JSON Booleans
`BoolAsInt`	:	`codec` $\mathbb{B}$	JSON Numbers 0 or 1
`StandardString`	:	`codec` $\mathbb{S}$	JSON Strings

- `StandardInt` decodes 1 into the float `1`, but 2^{54} into the string `"18014398509481984"`

Elliptic Curve Code Operators


```
{
  "degree": 1,
  "x-coordinates_of_integral_points": "[5,16]",
  "isogeny_matrix": [[1,5,25],[5,1,5],[25,5,1]],
  "label": "11a1",
  "_id": "ObjectId('4f71d4304d47869291435e6e')",
  ...
}
```


- Matrix in the `isogeny_matrix` field
  - $\begin{bmatrix} 1 & 5 & 25 \\ 5 & 1 & 5 \\ 25 & 5 & 1 \end{bmatrix}$
  - represented as `[[1,5,25],[5,1,5],[25,5,1]]`

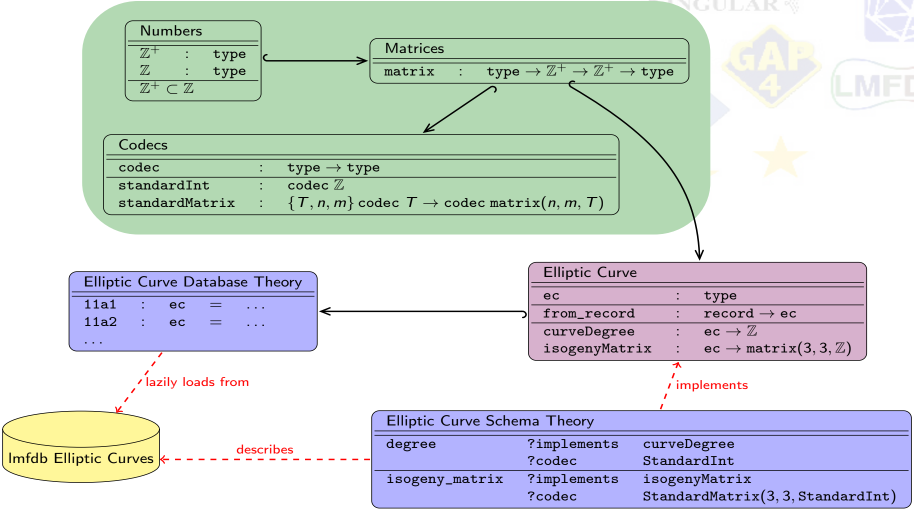
Codec Operator Examples

- **Definition 4.4 (Codec Operator).** A codec operator is a function which takes a codec, a set of parameters, and returns a codec.

Codecs (continued)			
`StandardList`	:	codec $T \to$ codec $\mathrm{List}(T)$	JSON list, recursively coding each element of the list
`StandardVector`	:	codec $T \to$ codec $\mathrm{Vector}(n, T)$	JSON list of fixed length n
`StandardMatrix`	:	codec $T \to$ codec $\mathrm{Matrix}(n, m, T)$	JSON list of n lists of length m

- `StandardMatrix`(`StandardInt`, 3, 3) generates the codec we used for the isogeny matrix

Our approach: Virtual Theories



An Example of a Query

- **Example 4.5.** Finding all cyclic transitive groups in LMFDB (recall from above)

```
x in (related to ( literal ‘lmfdb:db/transitivegroups?group ) by (object declares))
  | holds x (x cyclic x ∗=∗ true)
```

- This example does not rely on the internal structure of LMFDB
- can be translated into an LMFDB query using the just-defined codecs theory
- `http://www.lmfdb.org/api/transitivegroups/groups/?cyc=1`

Solving John's Hecke Fields Use Case

- Remember: John wanted to study number fields of HMFs via their Hecke field polynomials.
- John computes in SageMath and accesses LMFDB programmatically at the mathematical level (directly in the MitM dialect)
  - Build a query for LMFDB (no network connection with MitM server)

```
lmfdb = MitM.lmfdb
algebra = MitM.smglom.algebra

# a MitM expression that returns all hmf_forms with degree 2
hmfs_query = lmfdb.hmf_forms.where(algebra.base_field_degree(2))

# a MitM expression that additionally extracts the Hecke polynomial
# from each hmf_form
polys_query = hmfs_query.map(lambda x: lmfdb.hecke(x))
```

  - run the query via MitM and obtain the set of Sage polynomials

```
polys = MitM.run(polys_query)
```

  - further processing in Sage

```
fields = [NumberField(p) for p in polys]
```

. . . and the same in a Jupyter Notebook

- **Example 4.6.** John's use case in a Jupyter Notebook (with a SageMath kernel)

```
In [1]: # import all the relevant bits from MitM
        import MitM
        from MitM import lmfdb, algebra

In [6]: # put the query together
        query = lmfdb.hmf_hecke.where(
            algebra.HilbertNewforms.base_field_degree(int(2)),
            algebra.HilbertNewforms.dimension(int(2)),
        ).limit(until=int(10)).map(algebra.HeckeEigenvalues.heckePolynomial)

In [7]: # and run it
        MitM.run(query)

Out[7]: [x^2 + x + 7,
         x^2 + x + 4,
         x^2 + x + 7,
         x^2 + x + 2,
         x^2 + x + 4,
         x^2 + 12,
         x^2 + x + 1,
         x^2 + x + 4,
         x^2 + x + 2,
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In [ ]:
```

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         x^2 + x + 1]

In [ ]:
```

Upshot: We have a programmatic, math-level API for LMFDB

- - embed into any MitM-connected system (syntax adapted to host system)
  - no DB-level JSON encodings, but concepts like HilbertNewForms.dimension.

5 Jupyter Integration into MathHub

MathHub: A Portal and Archive of Flexiformal Maths

- Idea: learn from the open source community, offer a code repository with management support that acts as a hub for publication/development projects.
- MathHub: a collaborative development/hosting/publishing system of open-source, formal/informal math. (See `http://mathhub.info`)

MathHub: A Portal and Archive of Flexiformal Maths

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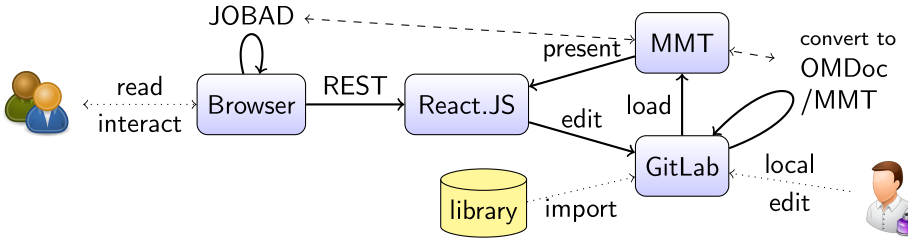
- MathHub: a collaborative development/hosting/publishing system of open-source, formal/informal math. (See `http://mathhub.info`)
- MathHub Architeture: Three core components (meet requirements above)
  - Representation: *OMDoc*/MMT mechanized by the MMT system.
  - Repositories: GitLab (git-based public/private repositories)
  - Front-End: React.JS (all content served by MMT)



An OpenDreamKit Risk come True

- Drupal Apocalypse:
  - The MathHub front-end was based on Drupal (very popular, mature)
  - our Drupal server was repeatedly hacked and compromised $\leadsto$ large maintenance overhead
- Decision in April 2018: Completely re-develop MathHub front-end using a web framework only.
  This was planned anyway (Drupal too heavyweight), but cost us months developer time until now.
- The new architecture (Docker compose + JSON + React.JS) helped integrate with Jupyter.

KPIs and Deliverables for WP6

- MitM-connected Systems: four (GAP, Sage, LMFDB, Singular) (See D6.5)
- Formal MitM Ontology: 55 files, 2600 LoF, 360 commits (See D6.8)
- Informal MitM Ontology: 815 theories, 1700 concepts in English, German, (Romanian, Chinese)
- MitM System API Theories (GAP, Sage, LMFDB, Singular): 1.000+ Theories, 22.000 Concepts.
- Multi-Site involvement of Researchers (Mobility of Researchers)
  - PD. Dr. Florian Rabe (Joint appointment UPSud/FAU)
  - Felix Schmoll Summer Internship (From JacU to St.Andrews)
  - Prof. Nathan Carter (Bentley Univ.) in St. Andrews (Sabbatical)
- Heavy interest by the theorem proving community about MitM Ontology
- Logipedia (`http://logipedia.science`) adopts the MitM principle of integrating (logical) systems by aligning concepts.
- First ODK-external MitM "user" for the next months: Andrea Thevis, Saarbrücken

Conclusion: What are we doing in WP6 in terms of a VRE

- SageMath/CoCalc and WP6 approach (Math-in-the-Middle; MitM) are both attempts at making a VRE Toolkit.
- SageMath/CoCalc is very successful, because integration is lightweight:
  - It makes no assumption on the meaning of math objects exchanged.
  - Restricts itself to master-slave integration of systems into SageMath.

  But there are safety, extensibility, and flexibility issues!
- MitM tries to take the high road (make possible by OpenDreamKit)
  - Safety: by semantic (i.e. context-aware) objects passed.
  - Extensibility: any open-API system (i.e. with API CDs) can play.
  - Flexibility: full peer-to-peer possibilities. (future: service discovery)

  But we have to develop a whole new framework!(Review 1 $\rightsquigarrow$ Proof of Concept)
- Review Period2: State of WP6 (MitM) Integration
  - Developed mathematical use-cases (what do researchers want to do)
  - Extended middleware, grown MitM ontology, collected alignments
  - Jupyter integration into MathHub.info
- Plan for Review Period 3: Extend to external use-cases users (scale and deploy publicly)
- overall pattern: design – prototype – scale

References I