EXPERIMENTAL MATHEMATICS

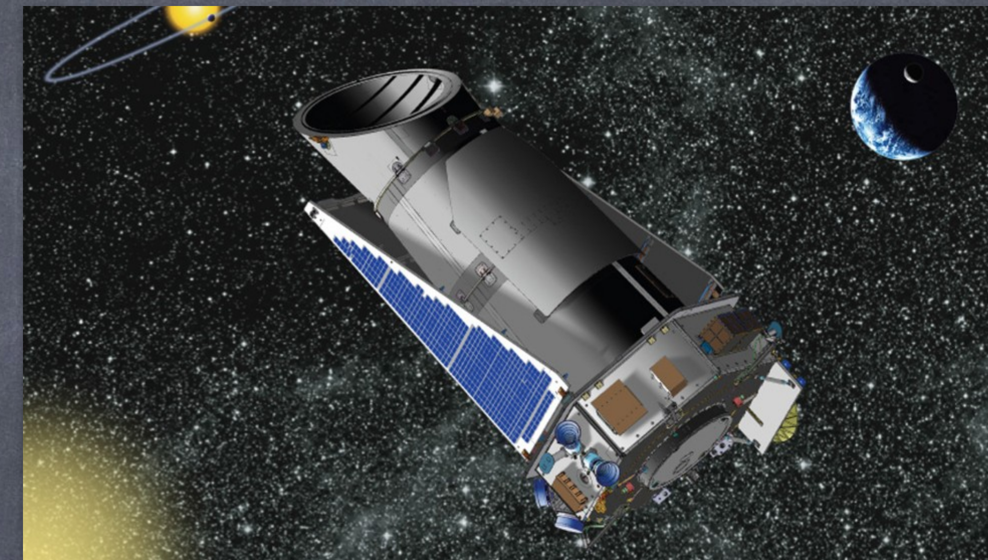
François Bergeron, LaCIM

EXPERIMENTAL MATHEMATICS

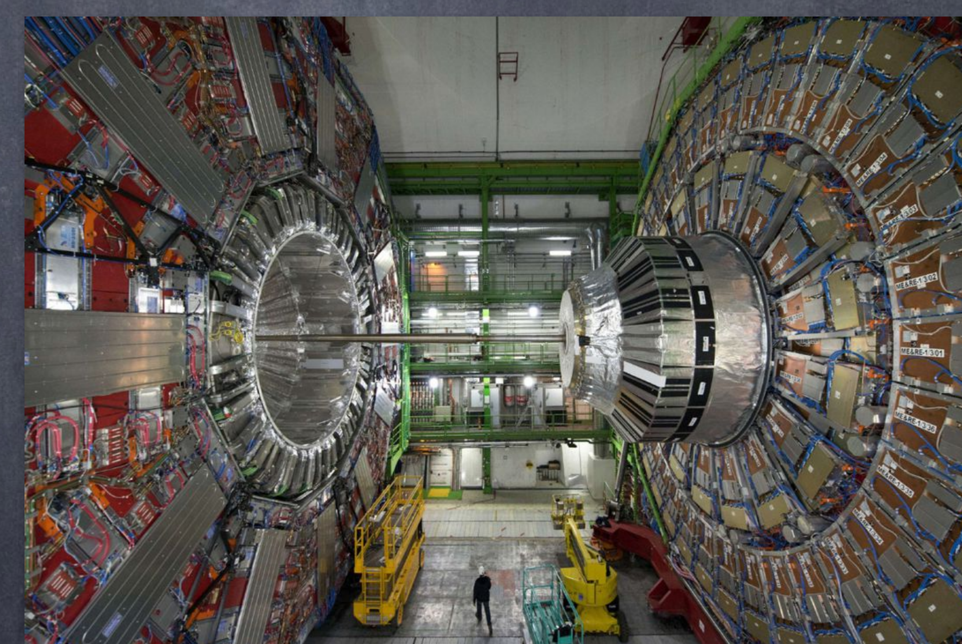
AIM:

- FIND FORMULAS
- FIND IDENTITIES
- FIND THEOREMS

HELP TO DISCOVERY



KEPLER SPACE TELESCOPE



LARGE HADRON COLLIDER

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A Long Tradition

THE TWO MOST IMPORTANT TOOLS FOR RECOGNIZING NEW INTERESTING MATHEMATICS

BEAUTY & SURPRISE

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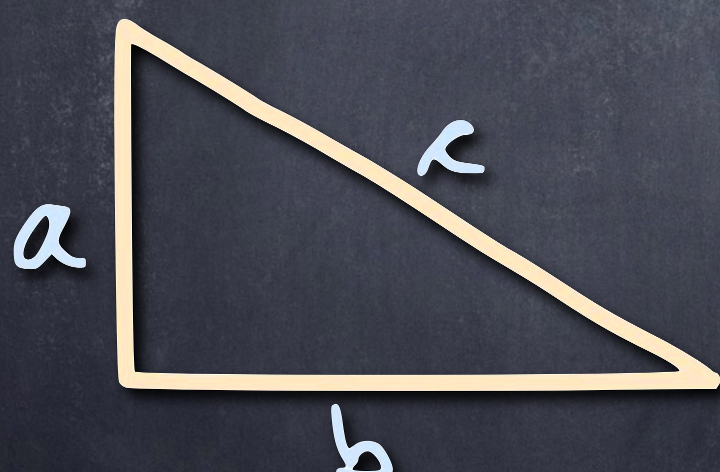
$$e^{i\pi}+1=0$$

$$|G| = \sum_{\lambda} d_{\lambda}^{2}$$

$$\sum_{n\geq 1} \frac{1}{n^{s}} = \prod_{p \text{ PRIME}} \left(1-\frac{1}{p^{s}}\right)$$

$$s_n(x\cdot y) = \sum_{\mu\vdash n} s_\mu(x)\, s_\mu(y)$$

$$\int_a^b f'(x)\,dx = f(b)-f(a)$$



$$a^2+b^2=c^2$$

$$\frac{\pi}{4} = 1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\cdots$$

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The Best Programming Language:

Mathematics

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MATHEMATICS AS A PROGRAMMING LANGUAGE

- WITH FUNCTIONS
- WITH SERIES
- WITH IDEALS
- ETC.

EXAMPLE

- WITH SERIES

$\alpha = (a_1 \dots a_m)$ $\beta = (b_1 \dots b_n)$

m_{ij}, a_i, b_j in $\mathbb{N}$

$$\mathcal{M}_{\alpha,\beta} := \Big\{ M = (m_{ij}) \,\Big|\, \sum_j m_{ij} = a_i \quad \text{AND} \quad \sum_i m_{ij} = b_j \Big\}$$

$$\begin{array}{c}
\begin{array}{cc}
\begin{pmatrix}
m_{11} & m_{12} & \cdots & m_{1n} \\
m_{21} & m_{22} & \cdots & m_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
m_{m1} & m_{m2} & \cdots & m_{mn}
\end{pmatrix} &
\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_m \end{array}
\end{array} \\
\begin{array}{cccc} b_1 & b_2 & \cdots & b_n \end{array}
\end{array}$$

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EXAMPLE

- WITH SERIES

$$S := \prod_{1\le i,j\le n} \frac{1}{1-z_{ij}\,x_i\,y_j}$$

$$\sum_{M\in \mathcal{M}_{\alpha,\beta}} z^M = \mathrm{COEFF}\left(S, x^\alpha y^\beta\right)$$

$$x^\alpha = x_1^{a_1}\cdots x_n^{a_n} \qquad y^\beta = y_1^{b_1}\cdots y_n^{b_n}$$

$$z^M = \prod_{M=(m_{ij})} z_{ij}^{m_{ij}}$$

COMPUTER ALGEBRA SYSTEMS SHOULD

- HAVE THE GENERAL MATH USER IN MIND.
- CONVINCE THE SKEPTICAL MATH ADEPT
- IDEALLY:
  - CODE THAT IS EASY TO READ WITH NO SPECIAL PREPERATION
  - CLOSE TO MATHEMATICAL PRACTICE
  - OUTPUT VERY MATH-LIKE
  - EASY TO MANIPULATE

COMPUTER ALGEBRA SYSTEMS SHOULD

- IDEALLY:
  - CODE THAT IS EASY TO READ WITH NO SPECIAL PREPERATION
  - CLOSE TO MATHEMATICAL PRACTICE
  - OUTPUT VERY MATH-LIKE
  - EASY TO MANIPULATE
  - MAKE RECURSIVITY EASY (OPTION FORGET)
  - SHOULD NOT ASSUME ELABORATE COMPUTER SKILL

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- INTERACTIVE MATH TEXT DOCUMENTATION
- EXAMPLES ARE BEST
- MATH IS THE BEST CODE
- MATHEMATICAL DESCRIPTION OF FUNCTIONS & ALGORITHMS
- SOURCE "CODE" MATHEMATICAL

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Hands on Mathematics

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To series

$$\prod_{k\geq 1} \boxed{\frac{1}{1-x_k}}$$

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$$\prod_{k\geq 1} \frac{1}{1-x_k} = \prod_{k\geq 1} \sum_{n\geq 0} x_k^n$$

$$\prod_{k\geq 1}\frac{1}{1-x_k} = 1 + (x_1 + x_2 + \dots)$$
$$+ (x_1^2 + x_2^2 + \dots + x_1 x_2 + \dots)$$
$$\dots$$

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$$\prod_{k\geq 1} \frac{1}{1-x_k} = \sum_{n\geq 0} h_n$$

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$$\sum_{n\geq 0} h_n = \prod_{k\geq 1} \frac{1}{1-x_k}$$

$$\sum_{n\geq 0} h_n = \prod_{k\geq 1} \boxed{\frac{1}{1-x_k}}$$

To exp

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$$\sum_{n\geq 0} h_n = \prod_{k\geq 1} \exp\left(\log \frac{1}{1-x_k}\right)$$

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$$\sum_{n\geq 0} h_n = \prod_{k\geq 1} \exp\left(\boxed{\log \frac{1}{1-x_k}}\right)$$

To series

$$\sum_{n\geq 0} h_n = \prod_{k\geq 1} \exp\Big(\sum_{j\geq 1} x_k^j/j\Big)$$

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$$\sum_{n\geq 0} h_n = \exp \sum_{k\geq 1} \sum_{j\geq 1} x_k^j/j$$

$$\sum_{n\geq 0} h_n = \exp \sum_{j\geq 1} \sum_{k\geq 1} x_k^j/j$$

$$\sum_{n\geq 0} h_n = \exp \sum_{j\geq 1} \frac{1}{j} p_j$$

$$p_j = \sum_{k\geq 1} x_k^j$$

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USING NOTEBOOKS

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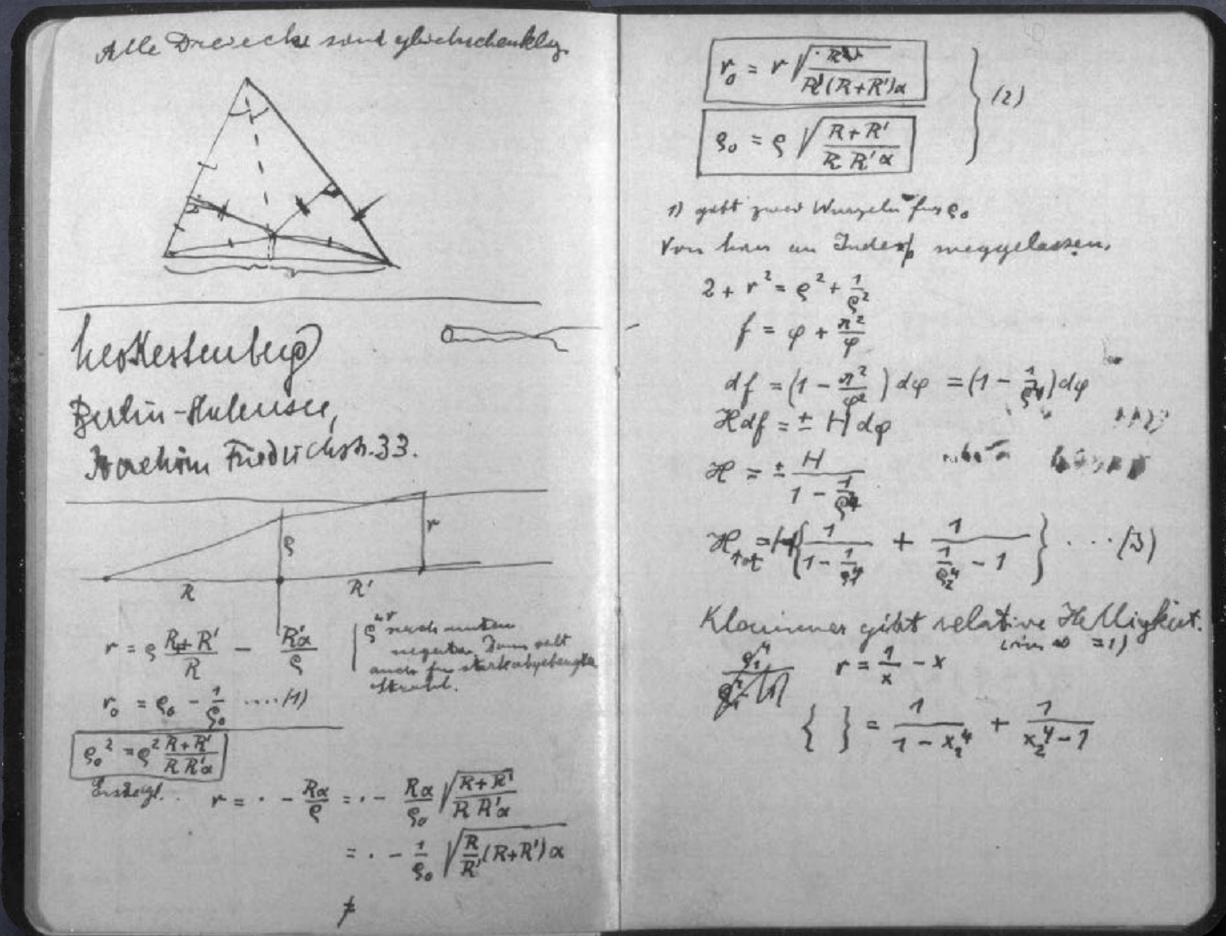

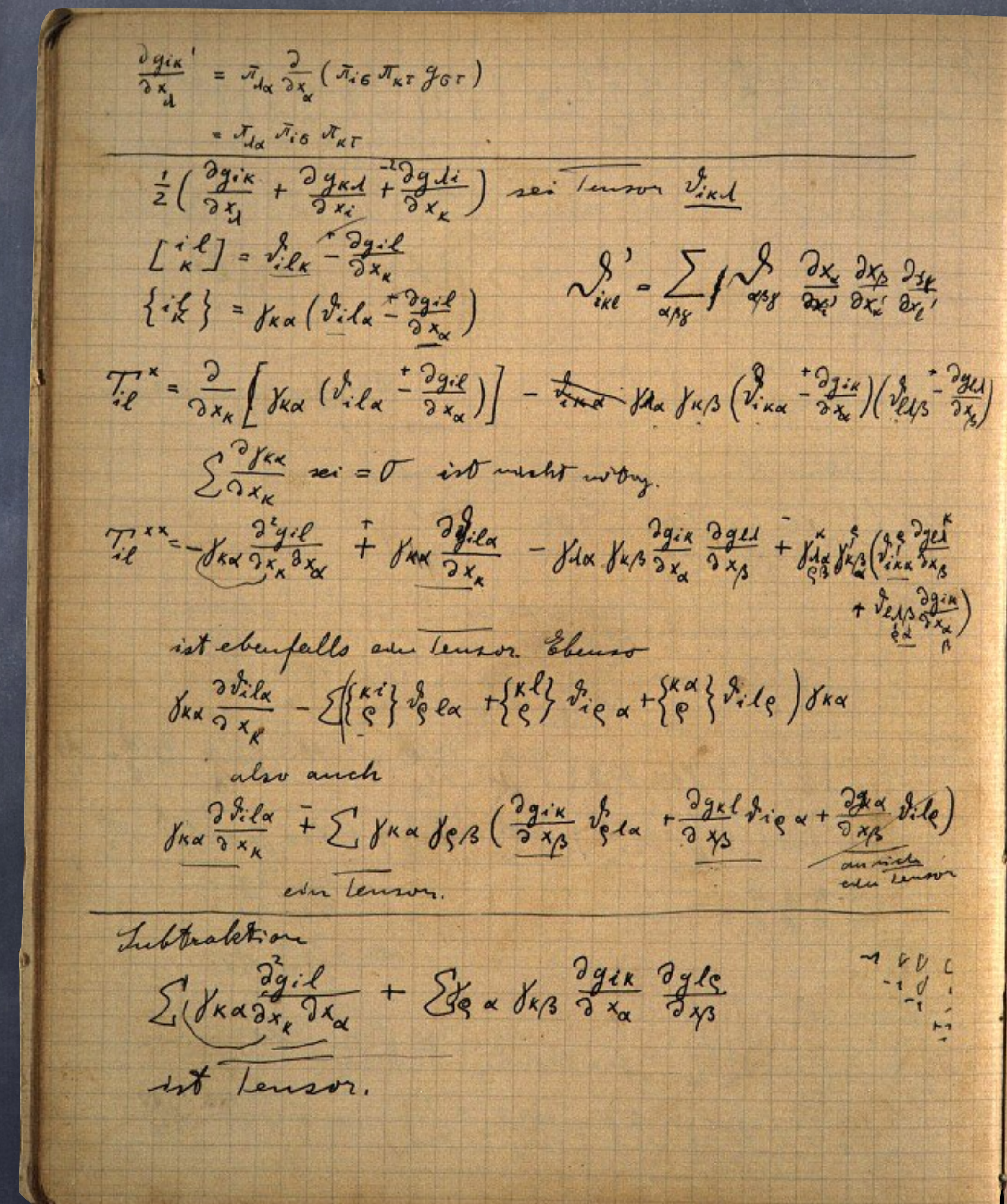

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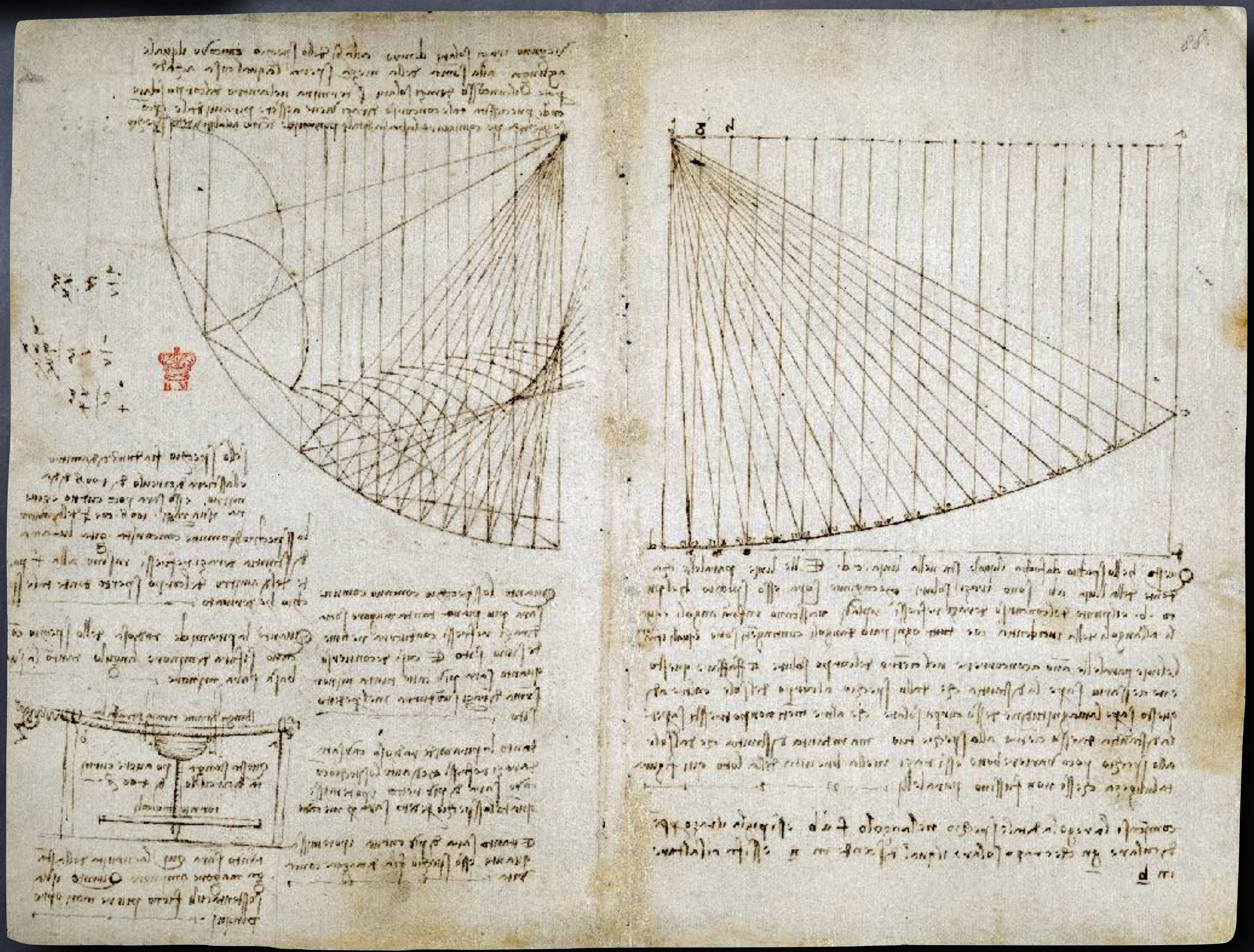

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jupyter Symmetric-Functions Last Checkpoint: 2018-12-10 (unsaved changes)

Logout

File Edit View Insert Cell Kernel Widgets Help

Not Trusted

SageMath 8.6

Run Markdown

Schur Positivity

When computing with symmetric functions, one often wants to check a given symmetric function is Schur positive or not. In our current setup, this means that coefficients polynomials in $\mathbb{N}[q,t]$. The following function returns `True` if the given symmetric function is Schur positive and `False` if not.

In [ ]:

```
F = s([4,1])+s([3,2])
print(F.is_schur_positive())
```

Schur positivity is a rare phenomena in general, but symmetric functions that come from representation theory are Schur positive. One can show that the probability that a degree n monomial positive is Schur positive is equal to

$$\prod_{\mu\vdash n} \frac{1}{k_\mu}, \qquad \text{where} \qquad k_\mu := \sum_{\nu\vdash n} K_{\mu,\nu},$$

with $K_{\mu,\nu}$ the **Kostka numbers**. Recall that these occur in the expansion of the Schur functions in terms of the monomial functions:

$$s_\mu = \sum_\nu K_{\mu,\nu}\, m_\nu .$$

define in sage

In [4]:

```
def K(mu,nu):
    return s(mu).scalar(h(nu))

def k(mu):
    n=add(j for j in mu)
    return add(K(mu,nu) for nu in Partitions(n))

def prob_Schur_positive(n): return 1/mul(k(mu) for mu in Partitions(n))
```

Rareness of Schur-positivity is then demonstrated by the values:

In [5]:

```
show([prob_Schur_positive(n) for n in range(1,10)])
```

$$\left[1, \frac{1}{2}, \frac{1}{9}, \frac{1}{560}, \frac{1}{480480}, \frac{1}{1027458432000}, \frac{1}{2465474364698304960000}, \frac{1}{5037877939056430776561153702706544640000}, \frac{1}{10676427975710573489340578264279464205105289199689400320000000000}\right]$$

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SCHUR-POSITIVITY is RARE

F.B.

Vic Reiner

Rebecca Patrias

$\boxed{\text{THM}}$ THE PROBABILITY THAT A MONOMIAL POSITIVE SYMMETRIC FUNCTION IS SCHUR POSITIVE IS:

$$\prod_{\mu \vdash d} \Big(\sum_{\lambda} K_{\lambda\mu} \Big)^{-1}$$

SURPRISE

Schur Positivity

- Representation theory of $\mathbb{S}_n$
- Representation theory of GL_n
- Algebraic geometry
- Combinatorics
- Geometric complexity theory

Beauty & surprise

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Guessing Tools

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- Guess a recurrence for the sequence $1, 1, 2, 5, 14, 42, 132, \ldots$

$$(n+1)\, a_n = (4n-2)\, a_{n-1},$$

$$a_0 = 1$$

- Guess a recurrence for the sequence $a, b, ad+bc, acd+bc^2+bd,$

$$ac^2d + ad^2 + bc^3 + 2bcd, \ldots$$

- GUESS A RECURRENCE FOR THE SEQUENCE $1, 1, 2, 5, 14, 42, 132, \ldots$

$$(n+1)\, a_n = (4n-2)\, a_{n-1},$$

$$a_0 = 1$$

- GUESS A RECURRENCE FOR THE SEQUENCE $a,\ b,\ ad+bc,\ acd+bc^2+bd,$

$$ac^2d + ad^2 + bc^3 + 2bcd, \ldots$$

- GUESS A RECURRENCE FOR THE SEQUENCE $a, b, ad+bc, acd+bc^2+bd, ac^2d+ad^2+bc^3+2bcd, \ldots$

$$f_n = c\cdot f_{n-1} + d\, f_{n-2},$$

$$f(0)=a \quad , \quad f(1)=b$$

- GUESS THE GENERATING FUNCTION OF THE SEQUENCE

$1, 2, 4, 7, 11, 16, 22, \ldots$

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- GUESS A RECURRENCE FOR THE SEQUENCE $a, b, ad+bc, acd+bc^2+bd, ac^2d+ad^2+bc^3+2bcd, \ldots$

$$f_n = c\cdot f_{n-1} + d\, f_{n-2},$$

$$f(0) = a \quad , \quad f(1) = b$$

- GUESS THE GENERATING FUNCTION OF THE SEQUENCE

$1, 2, 4, 7, 11, 16, 22, \ldots$

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- Guess the generating function of the sequence

$1, 2, 4, 7, 11, 16, 22, \ldots$

$$\frac{x^2 - x + 1}{(1-x)^3}$$

- Guess the generating function of the sequence

$1, 1, 3, 10, 41, 196, 1057, \ldots$

$$e^{xe^x}$$

FINDING ALGEBRAIC SOLUTIONS OF DIFFERENTIAL EQUATIONS.

```
> {y(x) = -1+(1-9*x)*(diff(y(x), x))+(1/6)*x*(4-27*x)*(diff(y(x), x, x)),y(0)=0};
```

$$\left\{y(0)=0,\, y(x)=-1+(1-9\,x)\,y'(x)+\frac{x\,(4-27\,x)\,y''(x)}{6}\right\}$$

```
> Series_solve(%);
```

$$x+3\,x^2+12\,x^3+55\,x^4+273\,x^5+1428\,x^6+7752\,x^7+43263\,x^8+246675\,x^9+1430715\,x^{10}+8414640\,x^{11}+50067108\,x^{12}+300830572\,x^{13}+1822766520\,x^{14}+O(x^{15})$$

```
> Series_to_algeq(%,f);
```

$$x+(3\,x-1)\,f+3\,x\,f^2+x\,f^3$$

```
> f=factor(%+f);
```

$$f=x\,(f+1)^3$$

```
>
```

A GUESS

CAN BE VERIFIED ALGORITHMICALLY.

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ABSTRACT IS BETTER

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 8 & 16 \\ 3 & 9 & 27 & 81 \\ 4 & 16 & 64 & 256 \end{pmatrix} \xrightarrow{\det} 2^5 \times 3^2$$

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ABSTRACT IS BETTER

$$\begin{pmatrix} a_1 & a_1^2 & a_1^3 & a_1^4 \\ a_2 & a_2^2 & a_2^3 & a_2^4 \\ a_3 & a_3^2 & a_3^3 & a_3^4 \\ a_4 & a_4^2 & a_4^3 & a_4^4 \end{pmatrix} \xrightarrow{\det} \begin{aligned} & a_1a_2a_3a_4 \\ & \times(a_4-a_1)(a_4-a_2)(a_4-a_3) \\ & \times(a_3-a_1)(a_3-a_2)(a_2-a_1) \end{aligned}$$

q-ANALOGS ARE BETTER THAN NUMBERS

$$[n]_q := 1 + q + q^2 + \cdots + q^{n-1}$$

$$n!_q := [1]_q [2]_q \cdots [n]_q$$

$$\begin{bmatrix} n \\ k \end{bmatrix}_q := \frac{n!_q}{k!_q\,(n-k)!_q}$$

$f(q) = \,?$ EXPRESSED IN TERMS OF $[n]_q$

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q-ANALOGS ARE BETTER THAN NUMBERS

$\begin{bmatrix} n \\ k \end{bmatrix}_q$ "ALL" DIFFERENT

(TAKING INTO ACCOUNT OBVIOUS SYMMETRIES)

$\binom{n}{k}$ NOT ALL DIFFERENT

(EVEN TAKING INTO ACCOUNT OBVIOUS SYMMETRIES)

$$\binom{22}{3} = \binom{56}{2}$$

$$\binom{14}{6} = \binom{15}{5} = \binom{78}{2}$$

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CAN YOU PROVE THAT:

FOR $a \leq c, d \leq b$ SUCH THAT $ab = cd$

$$\begin{bmatrix} a+b \\ a \end{bmatrix}_q - \begin{bmatrix} c+d \\ c \end{bmatrix}_q \in \mathbb{N}[q] \quad ?$$

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EXAMPLE

In [17]:
```
E5=Eval1(CalE_mn(5),q)
E5
```

Out[17]:

$$q^{10}s_{11111}+\left(q^9+q^8+q^7+q^6\right)s_{2111}+\left(q^8+q^7+q^6+q^5+q^4\right)s_{221}$$
$$+\left(q^7+q^6+2q^5+q^4+q^3\right)s_{311}+\left(q^6+q^5+q^4+q^3+q^2\right)s_{32}$$
$$+\left(q^4+q^3+q^2+q\right)s_{41}+s_5$$

In [14]:
```
E5.map_coefficients(to_qn)
```

Out[14]:

$$q^{10}s_{11111}+(q_4)\cdot q^6 s_{2111}+(q_5)\cdot q^4 s_{221}+\left(\frac{q_3q_4}{q_2}\right)\cdot q^3 s_{311}+(q_5)\cdot q^2 s_{32}+(q_4)\cdot q s_{41}$$

EXAMPLE

In [17]:
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E5=Eval1(CalE_mn(5),q)
E5
```

Out[17]:

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$[n]_q$ CODED AS q_n

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EXPERIMENTAL MATHEMATICS

VOL. 1, 1992 No. 4

Computing the Generating Function of a Series Given Its First Few Terms

François Bergeron and Simon Plouffe

CONTENTS

We outline an approach for the computation of a good candidate for the generating function of a power series for which only the first few coefficients are known. More precisely, if the derivative, the logarithmic derivative, the reversion, or another transformation of a given power series (even with polynomial coefficients) appears to admit a rational generating function, we compute the generating function of the original series by applying the inverse of those transformations to the rational generating function found.

1. INTRODUCTION

We address the problem of finding the generating function $f(x)$ of a power series

$$\alpha(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots,$$

of which we know only a limited number of initial terms. We say that $\alpha(x)$ has *precision* n if all coefficients up to x^n are known. Clearly, in the absence of additional information, the knowledge of $\alpha(x)$ to any finite precision is not sufficient to determine $f(x)$ uniquely.

HOLONOMY PARADIGM ⟷ D-FINITE

ORE ALGEBRAS (OPERATORS)

GRÖBNER BASIS (SKEW POLYNOMIALS)

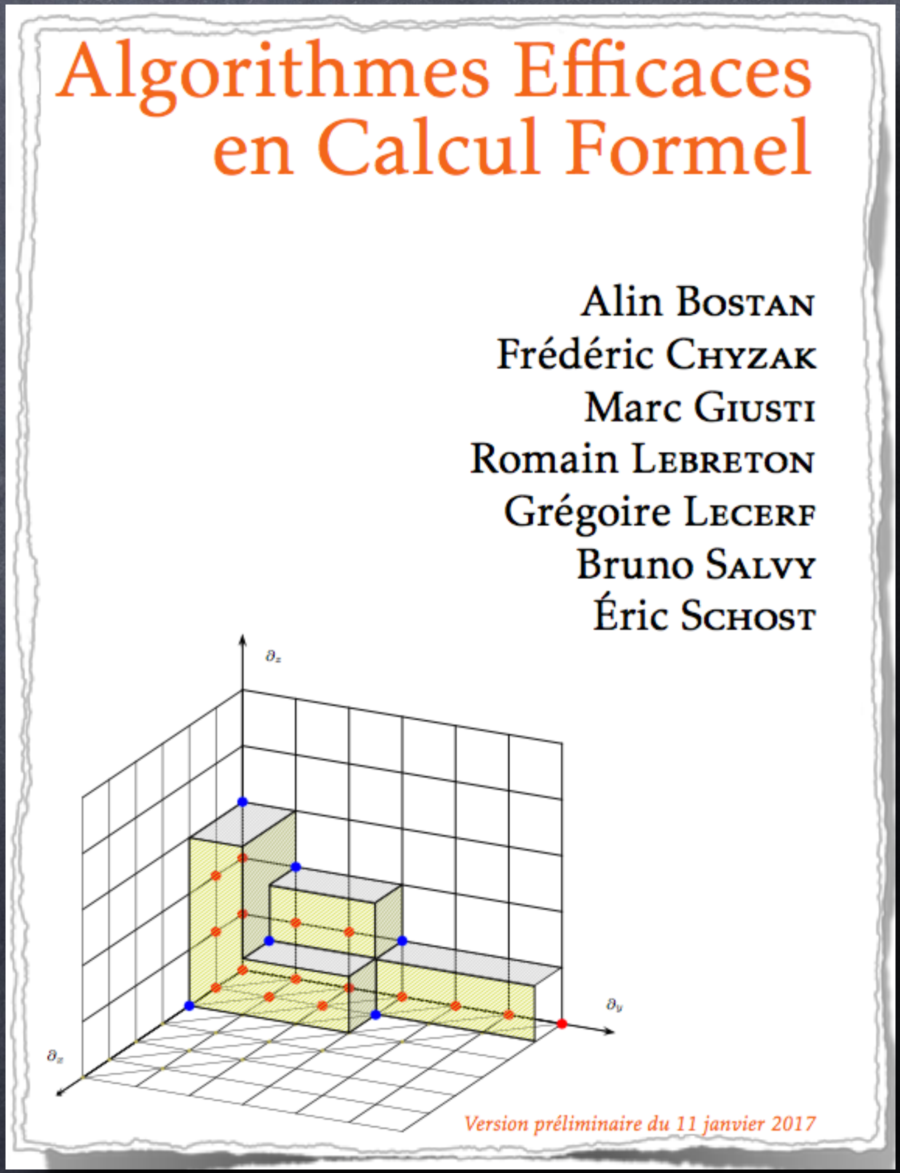
MGFUN (CHYZAK)

Europ. J. Combinatorics (1980) **1**, 175–188

Differentiably Finite Power Series

R. P. STANLEY*

A formal power series $\sum f(n)x^n$ is said to be differentiably finite if it satisfies a linear differential equation with polynomial coefficients. Such power series arise in a wide variety of problems in enumerative combinatorics. The basic properties of such series of significance to combinatorics are surveyed. Some reciprocity theorems are proved which link two such series together. A number of examples, applications and open problems are discussed.



EXAMPLES OF OPEN PROBLEMS

CONJECTURES

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Dimension of k-variate diagonal coinvariant spaces in k*m variables

```
(dim(F[1]))(k) = 1
(dim(F[2]))(k) = k+1
(dim(F[3]))(k) = 1+2*k^2+(17/6)*k+(1/6)*k^3
(dim(F[4]))(k) = 1+(103/16)*k^3+(439/45)*k^2+(329/60)
*k+(19/240)*k^5+(179/144)*k^4+(1/720)*k^6
(dim(F[5]))(k) = (629/70)*k+1+(2563693/60480)*k^3+
(730759/25200)*k^2+(206293/172800)*k^6+(2187/256)*
k^5+(653297/22680)*k^4+(11/241920)*k^9+(3313/40320)*
k^7+(341/120960)*k^8+(1/3628800)*k^10
```

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Dimension of k-variate diagonal coinvariant spaces in k×m variables

```
(dim(F[1]))(k) = 1
(dim(F[2]))(k) = k+1
(dim(F[3]))(k) = 1+5*k+5*binomial(k, 2)+binomial(k, 3)
(dim(F[4]))(k) = 1+23*k+78*binomial(k, 2)+96*binomial(k,
3)+51*binomial(k, 4)+12*binomial(k, 5)+binomial(k, 6)
(dim(F[5]))(k) = 1+119*k+1057*binomial(k, 2)+3383*
binomial(k, 3)+5418*binomial(k, 4)+4949*binomial(k, 5)
+2733*binomial(k, 6)+926*binomial(k, 7)+188*binomial(k,
8)+21*binomial(k, 9)+binomial(k, 10)
```

$$\dim(F_m)(k)$$

Dimension of k-variate diagonal coinvariant spaces in $k \times m$ variables

$$dim(F_1)(k) = 1$$

$$dim(F_2)(k) = k + 2$$

$$dim(F_3)(k) = 6 + 10\,k + 6\binom{k}{2} + \binom{k}{3}$$

$$dim(F_4)(k) = 24 + 101\,k + 174\binom{k}{2} + 147\binom{k}{3} + 63\binom{k}{4} + 13\binom{k}{5} + \binom{k}{6}$$

$$dim(F_5)(k) = 120 + 1176\,k + 4440\binom{k}{2} + 8801\binom{k}{3} + 10367\binom{k}{4} + 7682\binom{k}{5} + 3659\binom{k}{6} + 1114\binom{k}{7} + 209\binom{k}{8} + 22\binom{k}{9} + \binom{k}{10}$$

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$$\dim(F_m)(0) = m!$$

DIMENSION OF k-VARIATE DIAGONAL COINVARIANT SPACES IN k×m VARIABLES

$$dim(F_1)(k) = 1$$

$$dim(F_2)(k) = k + 2$$

$$dim(F_3)(k) = 6 + 10\,k + 6\binom{k}{2} + \binom{k}{3}$$

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$$dim(F_m)(1) = (m+1)^{m-1}$$

DIMENSION OF k-VARIATE DIAGONAL COINVARIANT SPACES IN $k \times m$ VARIABLES

$$dim(F_1)(k) = 1$$

$$dim(F_2)(k) = k + 2$$

$$dim(F_3)(k) = 6 + 10\,k + 6\binom{k}{2} + \binom{k}{3}$$

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$$\dim(F_m)(2) = 2^m(m+1)^{m-2}$$

DIMENSION OF k-VARIATE DIAGONAL COINVARIANT SPACES IN k×m VARIABLES

CONJECTURES

$$dim(F_1)(k) = 1$$

$$dim(F_2)(k) = k + 2$$

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$$\dim(F_m)(3) = \;?$$

DIMENSION OF k-VARIATE DIAGONAL COINVARIANT SPACES IN $k \times m$ VARIABLES

CONJECTURES

$$dim(F_1)(k) = 1$$

$$dim(F_2)(k) = k + 2$$

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$$dim(F_5)(k) = 120 + 1176\,k + 4440\binom{k}{2} + 8801\binom{k}{3} + 10367\binom{k}{4} + 7682\binom{k}{5} + 3659\binom{k}{6} + 1114\binom{k}{7} + 209\binom{k}{8} + 22\binom{k}{9} + \binom{k}{10}$$

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$$dim(F_m)(-3) = \frac{(-1)^{m-1}}{m}\binom{2(m-1)}{m-1}$$

DIMENSION OF k-VARIATE DIAGONAL COINVARIANT SPACES IN $k \times m$ VARIABLES

CONJECTURES

$$dim(F_1)(k) = 1$$

$$dim(F_2)(k) = k + 2$$

$$dim(F_3)(k) = 6 + 10\,k + 6\binom{k}{2} + \binom{k}{3}$$

$$dim(F_4)(k) = 24 + 101\,k + 174\binom{k}{2} + 147\binom{k}{3} + 63\binom{k}{4} + 13\binom{k}{5} + \binom{k}{6}$$

$$dim(F_5)(k) = 120 + 1176\,k + 4440\binom{k}{2} + 8801\binom{k}{3} + 10367\binom{k}{4} + 7682\binom{k}{5} + 3659\binom{k}{6} + 1114\binom{k}{7} + 209\binom{k}{8} + 22\binom{k}{9} + \binom{k}{10}$$

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$$\dim(F_m)(-3) = \frac{(-1)^{m-1}}{m}\binom{2(m-1)}{m-1}$$

DIMENSION OF k-VARIATE DIAGONAL COINVARIANT SPACES IN $k \times m$ VARIABLES

CONJECTURES

$$alt_1(k) = 1$$

$$alt_2(k) = k + 1$$

$$alt_3(k) = 1 + 4\,k + 4\binom{k}{2} + \binom{k}{3}$$

$$alt_4(k) = 1 + 13\,k + 41\binom{k}{2} + 54\binom{k}{3} + 34\binom{k}{4} + 10\binom{k}{5} + \binom{k}{6}$$

$$alt_5(k) = 1 + 41\,k + 316\binom{k}{2} + 1038\binom{k}{3} + 1854\binom{k}{4} + 1991\binom{k}{5} + 1333\binom{k}{6}$$
$$+ 553\binom{k}{7} + 136\binom{k}{8} + 18\binom{k}{9} + \binom{k}{10}$$

DIAGONAL COINVARIANT SPACES in $k \times m$ VARIABLES

$$F_1(k) = e_1$$

$$F_2(k) = k\,e_2 + e_1^2$$

$$F_3(k) = e_1^3 + \left(3\,k + \binom{k}{2}\right) e_1\,e_2 + \left(k + 3\binom{k}{2} + \binom{k}{3}\right) e_3$$

$$F_4(k) = e_1^4 + \left(6\,k + 4\binom{k}{2} + \binom{k}{3}\right) e_1^2\,e_2 + \left(4\,k + 18\binom{k}{2} + 19\binom{k}{3} + 8\binom{k}{4} + \binom{k}{5}\right) e_1\,e_3 + \left(2\,k + 7\binom{k}{2} + 5\binom{k}{3} + \binom{k}{4}\right) e_2^2 + \left(k + 12\binom{k}{2} + 29\binom{k}{3} + 25\binom{k}{4} + 9\binom{k}{5} + \binom{k}{6}\right) e_4$$

Fin